Ownership Structure and Synergistic Takeover: Implications on Corporate Governance*

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Abstract

This paper explores how the managerial disciplinary role of synergistic takeover, initiated by bidders who seek the business synergy, interacts with the ownership structure of target firms. We model a firm in which the manager has private information about the state of economy and may disguise her under-provision of effort by misreporting the state to shareholders. The presence of bidders searching for synergies helps alleviate this agency problem, since the managerial misreport makes the firm undervalued and, thus, attracts more takeover bids. Our analysis shows that, while the control premium required by controlling shareholders reduces the likelihood of takeover incidence, it rather strengthens the managerial disciplinary effect of takeover in high growth firms that tend to provide large business synergies to potential acquirers. The analysis provides novel predictions on the correlation between the firm’s growth options, ownership structure and other governance mechanisms, such as managerial compensation and monitoring function of large shareholders, and offer policy implications regarding the social optimality of ownership concentration.

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1 Introduction

Optimal corporate ownership structure has been a longstanding question in corporate governance literature. While large controlling shareholders can address managerial agency problems by monitoring management and alleviating the free-riding problem in takeover (Grossman and Hart 1980, Shleifer and Vishny 1986), they may also expropriate other stakeholders by influencing management or deter efficient takeover to maintain their private control benefits (Stulz 1988, Bebchuk 1999). Empirical evidence about the effect of controlling shareholders, e.g., founding family, on the firm performance is also inconclusive.\(^1\) Amid the ongoing debate, this paper provides a new perspective about the role of controlling shareholders in the market disciplinary mechanism, and how it interacts with the firm’s growth options and other internal governance mechanisms.

This paper innovates the literature by considering a managerial disciplinary effect of synergistic takeover (e.g., vertical or horizontal integration), initiated by bidders who seek the business synergy from acquiring the target firm.\(^2\) This market disciplinary mechanism is practically relevant, and, more importantly for our research purpose, it complements the managerial monitoring function of controlling shareholders who may lack operational expertise relative to the bidders.\(^3\) Our analysis shows that the managerial disciplinary effect of synergistic takeover is affected by both the firm’s growth option (i.e., potential business synergy with other firms) and the control premium required by controlling shareholders, and, surprisingly, the control benefit can strengthen, not weaken, the disciplinary effect in high growth firms.

Why does the private benefit of controlling shareholders, which increase the takeover pre-

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\(^1\) See, e.g., Bertrand and Schoar (2006) who review empirical studies about the family ownership.


\(^3\) The managerial agency problem is relevant even in family firms. Previous studies report that a majority of family firms are managed the professional CEOs (e.g., Anderson and Reeb 2003a, Fahlenbrach 2009 Villalonga and Amit 2009). Bidders that operate in the same industry, for example, will be able to observe the state of the demand to assess the synergistic improvements. In contrast, the absence of heirs from the operational aspects of running the firm suggests their lack of operational expertise vis-a-vis the manager.
mium, *strengthen* the market disciplinary mechanism? To answer the question intuitively, suppose that the manager has no anti-takeover defense. In this case, the manager can secure herself from the takeover threats only by increasing the market value of the firm, and, therefore, the takeover threat can discipline the manager. In high growth firms, however, the manager may find it too costly to increase the market value enough to deter the synergistic takeover. The control premium required by controlling shareholders can complement the market disciplinary mechanism in this circumstance, and, specifically, it reduces the profitability of synergistic takeover and makes the acquirers’ bidding choice more sensitive to the current market value. That is, it allows the manager of high growth firms to reduce the takeover threat more significantly by increasing the market value. As shown below, the effect of ownership structure on the disciplining function of synergistic takeover also interacts with other governance mechanisms such as the managerial compensation and internal monitoring by controlling shareholders.

Formally, our model is built on the setting of Scharfstein (1988) who examines the managerial disciplinary effect of takeover threat. We consider an all-equity firm operated by a professional manager who does not hold ownership. The separation of ownership and control creates a managerial agency problem. Specifically, the manager acquires private information about the state of industry, either high (favorable) or low (unfavorable) state, before choosing her effort which is also unobservable to shareholders. Managerial effort is more productive in high state than in low state and, thus, shareholders are willing to induce higher effort in high state. Suppose that shareholders take all surplus and pay just enough to compensate for the cost of effort. Then, the manager would underreport high state as low state, and take the saved effort cost as information rent.

The agency problem can be mitigated by a market disciplinary mechanism. We consider potential acquirers who observe the state and assess the synergy from acquiring the firm. In the spirit of Scharfstein (1988), we assume that the incumbent manager is replaced after the takeover, and, thus, lose the opportunity to get information rent. The existence of takeover
threats disciplines the managerial misreporting since the undervalued firms attract more takeover attempts. The synergistic takeover, however, does not fully address the agency problem due to the possibility of negative business synergy that deters the acquirer from bidding for undervalued targets.

We next turn to the analysis of how the managerial disciplinary effect of synergistic takeover is associated with ownership structure. Our model considers two mutually exclusive ownership structures, namely, dispersed ownership and concentrated ownership, which differ in the presence of controlling shareholders (e.g., family owners). More specifically, we model controlling shareholders as those who have a private benefit of control (e.g., amenity potential of family owners) which does not destroy the firm value by itself. In the takeover market, the controlling shareholders require higher premium as a reward for giving up their control benefit, and, thus, reduce the incidence of takeovers relative to the dispersed ownership firms.

Notwithstanding their negative effect on the incidence of synergistic takeover, the controlling shareholders can strengthen managerial disciplinary effect of takeover in high growth firms that tend to offer acquirers large business synergies. Intuitively, in these firms, the increase in takeover premium exponentially reduces the probability of synergistic takeover, i.e., the small change in market value does not significantly affect the profitability of synergistic takeover. Therefore, the control premium required by controlling shareholders makes truthful managerial reporting (i.e., the market value enhancement by truthfully reporting high state) more effective in reducing the likelihood of takeover. Put it another way, the control premium increases the manager’s opportunity cost of misreporting, i.e., higher probability of a synergistic takeover and, in turn, it reduces the information rent taken by the

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4See Demsetz and Lehn (1985), Burkart et al. (2003), Almeida and Wolfenzon (2006), Ellul et al. (2010) for other papers that consider the private control benefit of large shareholders.


6Technically, our model shows that the sufficient condition for the complementarity of ownership concentration and market disciplinary mechanism is the log-convexity of the distribution function of potential business synergy.
manager. This intuition also implies that the existence of controlling shareholders diminishes the market disciplinary mechanism in mature firms that do not provide high synergies.

Our model analysis also provides a number of implications on the relationship between ownership structure and other governance mechanisms, such as managerial compensation and monitoring function of controlling shareholders. We show that disciplinary effect of synergistic takeover reduces the information rent paid to the manager and, thus, it diminishes the managerial incentive pay. This result implies that managerial pay-performance sensitivity is negatively associated with ownership concentration in growth firms.\(^7\) Furthermore, our analysis also shows that, in growth firms in which controlling shareholders strengthen the market disciplinary mechanism, monitoring function of controlling shareholders can also complement the market disciplinary mechanism, and, thus, ownership concentration increases the operating efficiency even more.\(^8\)

Finally, Our analysis also offers policy implications regarding the social optimality of concentrated ownership structure. Some regulators criticize the role of controlling shareholders in corporate governance and considers breaking up the concentrated ownership structure to facilitate the market for corporate control.\(^9\) Our analysis shows that the presence of controlling shareholders strengthen managerial disciplinary mechanism in high growth firms, and, furthermore, their internal monitoring functions can complement the market disciplinary mechanism in these firms. Though the control premium required by the controlling shareholders may deter even valuable takeovers, our findings suggest that regulators should approach very cautiously in dismantling concentrated ownership structure, in particular, in high growth firms. Our analysis also suggests that, if high growth firms are already owned by controlling shareholders, regulators may consider a policy that facilitates the market for

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\(^7\)Consistent with this prediction, Li et al. (2015) show that pay for performance sensitivity of professional CEOs is lower in firms with concentrated shareholders, and the effects are strongest in the firms controlled by families.

\(^8\)Using French data, Sraer and Thesmar (2007) find that outside CEOs tend to outperform among family firms. Li et al. (2015) show that pay-performance sensitivity of professional CEOs decreases with ownership concentration, and the effects are strongest in family-controlled firms.

\(^9\)E.g., the European Commission considers breaking up controlling ownership structures (Burkart and Panunzi 2006) to facilitate the market for corporate control.
corporate control in industry downturns to take advantage of internal monitoring.

Our paper contributes to the literature of ownership structure and agency problem in corporations. On one hand, controlling shareholders or managerial ownership alleviates the agency problems that arise from the separation of ownership and control (e.g., Shleifer and Vishny 1986). On the other hand, controlling shareholders expropriate other minor shareholders by influencing the management (e.g., Stulz 1988). Our model suggests that while controlling shareholders entrench the manager from takeover threat and reduce the minor shareholders’ value, it can mitigate managerial agency problem by complementing the disciplinary effect of synergistic takeover and enhance the firm value.

This paper also contributes to the literature that examines managerial disciplinary effect of takeover. Previous studies focus on the disciplinary takeovers such as Leveraged Buyout or hostile takeovers (e.g., Scharfstein 1988, Shleifer and Vishny 1997, Morck et al. 1988). Our analysis sheds light on the disciplinary effect of synergistic takeover. Due to the operational expertise of potential bidders, this market disciplinary mechanism can complement the monitoring functions of large shareholders. Furthermore, our analysis provides a new perspective on the relationship between the firm’s business characteristics and governance mechanisms: the market disciplinary mechanism is affected by the firm’s operational characteristics that determine their business synergy with other firms.

Finally, this paper also contributes to the literature that studies the effect of managerial entrenchment on the firm performance. While most papers focus on the firm value destruction by managerial entrenchment, an exception is Almazan and Suarez (2003) who show that managers may not put effort due to the concern that they will be replaced by better talented managers, and weak governance can be used as a commitment mechanism for the manager’s job security.\(^{10}\) Our analysis complements their finding and provides a new economic channel through which the managerial entrenchment can enhance the firm value.

The paper is organized as follows. Section 2 describes the model setup. Section 3 analyzes

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\(^{10}\)In the analysis of employee level, Sraer and Thesmar (2007) show that French family firms isolate employees from external shocks, and pay them less than non-family firms.
the case of dispersed ownership, and Section 4 presents the case of concentrated ownership, and compares it with dispersed ownership case. Section 5 concludes.

2 Model setup

We consider an all-equity firm that operates in a risk-neutral economy where the market rate of return is normalized to zero. The firm is run by a manager who has no wealth, limited liability and zero reservation utility. In this setting, we consider a synergistic takeover market in which firms engage in merger and acquisitions to gain operational synergies, and analyze how the possibility of synergistic takeover affects the managerial actions and also interacts with the ownership structure of the firm.

2.1 Firm, information, and ownership structure

The firm’s assets in place yield the cash flow net of costs, \( V = \theta eR \), determined by (i) a binary state of industry (e.g., industry-wise profitability shock) \( \theta \in \{ \theta_L, \theta_H \} \), where \( 0 < \theta_L < \theta_H \), (ii) the manager’s effort choice \( e \in [0, 1] \), and (iii) the revenue stream \( R > 0 \). While \( R \) and the realized net cash flow \( V \) are publicly observable in this setting, the realized state of industry \( \theta \) and the effort choice \( e \) is not observed by shareholders. Regarding \( \theta \), shareholders have a common prior that \( \theta = \theta_L \) with probability \( 1 - \eta \), and \( \theta_H \) with probability \( \eta \), while the manager observes the realized state before choosing \( e \). Regarding the choice of \( e \), the manager incurs a private cost \( C(e) = \frac{1}{2}ce^2 \).

To present our key intuition succinctly, we consider a binary ownership structure of the firm, denoted by \( k \in \{ C, D \} \), which stands for concentrated and dispersed ownership, respectively. The two ownership structure differs only in the existence of controlling shareholders (e.g., family owners). Specifically, dispersed ownership structure corresponds to the case in which shareholders are atomistic, while concentrated ownership structure contains controlling
shareholders who hold controlling blocks and gain a private benefit of control $B > 0$.\footnote{These benefits, e.g., the amenity potential of family firms, differ from the usual interpretation of private benefits that are directly at the expense of minority shareholders (see Demsetz and Lehn 1985).}

Notably, we abstract from several issues of the controlling shareholders to focus on our main results. First, in our baseline model, we do not consider the managerial monitoring function of controlling shareholders.\footnote{See, e.g., Anderson and Reeb (2003a), Anderson and Reeb (2003b) and Villalonga and Amit (2006) for the information advantage of founding families relative to other shareholders.} We relax this assumption and extend the model to explicitly consider the role of their managerial monitoring function in Section 4.2. Second, controlling shareholders gain the private benefit of control without expropriating the minor shareholders in the incumbent business. As shown below, however, the control benefit increases the takeover premium, and, thus, reduce the surplus taken by the minor shareholders from takeover bids.

### 2.2 Synergistic takeover and market disciplinary mechanism

In this setting, we consider the managerial disciplinary effect of synergistic takeover market in which firms seek opportunities to gain business synergies from acquiring other firms. In spirit of Scharfstein (1988), we focus on the case in which the firm faces the possibility of being offered takeover bids from potential acquirers, but do not consider extending takeover bids to other firms, i.e., the firm is always a potential target in the takeover market. Specifically, the firm provides potential acquirers with a business synergy $\pi \in [\underline{\pi}, \overline{\pi}]$ drawn from a distribution function $H(x) \equiv Prob(\pi < x)$, where $H(\underline{\pi}) = 0$ and $H(\overline{\pi}) = 1$, with a density function $h(x) > 0$. The distribution function $H(\cdot)$ is a common prior before the realization of $\pi$.

Following Scharfstein (1988), we consider a stylized model of takeover: first, the bidder observes the realized state of industry $\theta$ and the realized business synergy $\pi$ before extending an offer to the target firm; second, the incumbent manager leaves the target firm without any severance pay at the occurrence of takeover, and the acquirer manages it to maximize the firm’s value; finally, as detailed in section 2.3, the takeover price is determined by the managerial employment contract in which shareholders commit to selling the firm at no less
than a specified price.\textsuperscript{13} The bidders can buy the firm at the specified minimum price since it is publicly known.\textsuperscript{14}

This stylized setting, while simplifying our analysis, reflects a key feature of synergistic takeover markets. The acquirers in synergistic takeovers seek the business synergies from acquiring target firms and, thus, they are likely to have superior information about the target firm’s business environment (i.e., $\theta$) relative to the target firm’s shareholders including controlling shareholders who hire professional managers. As shown below, this informational advantage of acquirers allows even controlling shareholders to utilize the managerial disciplinary effect of synergistic takeover.

2.3 Contracts and sequence of events

The sequence of events unfolds as follows. At $t = 0$, shareholders offer a contract to the manager who in turn decide whether to accept it. By the generalized revelation principle (Myerson 1982), we focus on a direct revelation contract, $\Gamma = [W(\hat{\theta}, V), P(\hat{\theta})]$, where $\hat{\theta} \in \{\hat{\theta}_L, \hat{\theta}_H\}$ corresponds to the manager’s message sent to shareholders regarding $\theta$, $W(.)$ to the managerial compensation, and $P(.)$ is the minimum price at or above which shareholders commit to selling the firm to the takeover bidders.

At $t = 1$, the manager and potential acquirers observe $\theta$, and the manager reports $(\hat{\theta})$ to shareholders. At $t = 2$, the potential acquirers observe $\pi$ and buys the firm at $P(\hat{\theta})$ if it finds the deal profitable. If the firm is sold, the incumbent manager leaves and the acquirer chooses the effort $e$. Otherwise, the incumbent manager chooses $e$.\textsuperscript{15} At $t = 3$, cash flow is realized and contracts are fulfilled. The sequence of events are summarized in Figure 1.

\textsuperscript{13}In an unreported analysis, we also consider a case in which shareholders do not set a committed selling price at the contracting stage. The main results are qualitatively consistent with those in this setting if the acquirer takes all surplus arising from a takeover.

\textsuperscript{14}In our setting, the minimum price can be also inferred by the rational bidders if there exists a stock market where incumbent shareholders trade immediately after acquiring new information.

\textsuperscript{15}In our setting, the manager exerts effort after the takeover stage, and, thus, do not consider a possibility that the manager provides undereffort due to the concern of being fired after takeover. See Almazan and Suarez (2003) who examine how the severance pay and the board structure mitigate this managerial undereffort problem.
3 Dispersed ownership firms

In this section, we examine the firm owned by dispersed shareholders who do not have a private benefit of control. To clearly present the role of each governance mechanism, we start by analyzing the first-best case in which shareholders observe θ and, thus, the realized net cash flow \( V \) is perfectly informative about the managerial choice of effort \( e \), and then examine the case in which dispersed shareholders do not observe \( \theta \) and \( e \) without considering the possibility of takeover bids. Finally, we consider the full model with the possibility of takeover bids.

3.1 Benchmark 1: observable \( \theta \) (the first-best case)

In the analysis of the first best case, we do not consider the takeover market since it does not affect the expected value of net cash flow \( V \). If shareholders observe \( \theta \) at \( t = 1 \), they can infer the managerial effort choice \( e \) from the realized net cash flow \( V \), and, thus, do not have to pay information rent to the manager (i.e., they pay zero managerial wage for any undesirable effort choice). For each possible state \( \theta_i \), \( i = l, h \), shareholders determine the effort choice \( e_i \) and the managerial wage \( \omega_i \) that solve the following problem:

\[
\max_{\langle e_i, \omega_i \rangle} \mathbb{E}(V) = \eta(\theta_H e_H R - \omega_H) + (1 - \eta)(\theta_L e_L R - \omega_L),
\]

(1)
subject to

\[
\begin{align*}
\omega_L - \frac{1}{2} ce^2_L &\geq 0, \\
\omega_H - \frac{1}{2} ce^2_H &\geq 0,
\end{align*}
\]

(PCL
\(\text{FB}\))

(PCH
\(\text{FB}\))

where the constraints ensure that the manager does not choose zero effort given their limited liability (i.e., the wage should be nonnegative). Solving the problem, we find that the first-best effort level \(e_i^*\) and corresponding managerial wage \(\omega_i^*\) are determined as:

\[
e_i^* = \frac{\theta_iR}{c} \quad \text{and} \quad \omega_i^* = \frac{(\theta_iR)^2}{2c}. \quad (2)
\]

### 3.2 Benchmark 2: unobservable \(\theta\) and no takeover bid

We turn to the case in which dispersed shareholders do not observe \(\theta\). Before solving for the optimal contract, let us discuss the managerial agency problem that arises in this setting. Suppose that shareholders assume that the manager will truthfully report about the state (i.e., \(\hat{\theta} = \theta\)) and offer the first best wage in (2), i.e., after receiving \(\hat{\theta} = \hat{\theta}_i\) (i.e., \(i = l, h\)), shareholders pay \(\omega_i^*\) if \(V = \theta_i e_i^* R\), and zero, otherwise. Then, if the manager underreports the favorable state \(\theta_H\) as \(\hat{\theta} = \hat{\theta}_L\), she can receive the wage \(\omega_L^*\) by exerting effort

\[
e \equiv \frac{\theta_L}{\theta_H} e_L^* < e_L^*.
\]

Intuitively, shareholders cannot tell whether the low outcome is due to unfavorable state of industry or to the managerial shirking, and, given the first-best wage contract, the manager has incentives to shirk and underreport the state of industry. Formally, given the first-best wage contract, the manager gains strictly positive ex-post utility from misreporting:

\[
\omega_L^* - \frac{1}{2} ce^2 = e_L^* \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] > 0.
\]
That is, in contrast to the first-best case, the manager gains an information rent in a favorable state. In the presence of managerial agency problems, the optimal contract should balance the trade-off between reducing the information rent and increasing the managerial effort.

Now we formally solve for the optimal compensation contract $W(\hat{\theta}, V)$ in dispersed-ownership firms. Without loss of generality, we focus on the contract which, for each report $\hat{\theta}_i (i = l, h)$, pays non-zero wage $\omega_i$ only if a specified net cash flow $V_i$ is realized (i.e., pays nothing for any other net cash flows). Then, the dispersed shareholders’ problem can be written as:

$$\max_{\langle e_i, \omega_i \rangle} \mathbb{E}(V) = \eta(\theta_H e_H R - \omega_H) + (1 - \eta)(\theta_L e_L R - \omega_L),$$

subject to

$$\omega_L - \frac{1}{2} c e_L^2 \geq 0, \quad \text{ (PCL)}$$
$$\omega_H - \frac{1}{2} c e_H^2 \geq 0, \quad \text{ (PCH)}$$
$$\omega_L - \frac{1}{2} c e_L^2 \geq \omega_H - \frac{1}{2} c e^2, \quad \text{ (ICL)}$$
$$\omega_H - \frac{1}{2} c e_H^2 \geq \omega_L - \frac{1}{2} c e^2, \quad \text{ (ICH)}$$

where $e = \frac{\theta_H}{\theta_L} e_H$ and $e = \frac{\theta_L}{\theta_H} e_L$. The constraints (ICL) and (ICH) ensure that truthfully reporting $\theta$ is incentive compatible to the manager.

As in the standard adverse selection framework, (ICH) binds at the optimum:

$$\omega_H = \omega_L + \frac{1}{2} c (e_H^2 - e_L^2) + \frac{1}{2} c (e_H^2 - e^2),$$

where the last term on the right-hand side corresponds to the information rent paid to the manager in state $\theta_H$. That is, the firm must provide higher pay-performance sensitivity than the first-best wage scheme to induce the manager not to underreport $\theta_H$. Furthermore,
(PCL\textsubscript{DN}) also binds since, otherwise, shareholders can increase their value by reducing $\omega_L$:

$$\omega_L = \frac{1}{2} ce_L^2,$$

By plugging (4) and (5) into (3), we can rewrite the shareholders’ problem as:

$$\max_{\langle \epsilon_i \rangle} \mathbb{E}(V) = \eta(\theta_H e_H R - \frac{1}{2} ce_H^2) + (1 - \eta)(\theta_L e_L R - \frac{1}{2} ce_L^2) - \eta \frac{1}{2} ce_L^2 \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right].$$

Notice that, relative to the shareholders’ objective function in the first-best case, (6) has the last additional term which corresponds to the expected value of information rent foregone to the manager in state $\theta_H$. The first order conditions are:

$$\frac{\partial \mathbb{E}(V)}{\partial e_H} = \eta(\theta_H R - ce_H^{DN}) = 0,$$

and

$$\frac{\partial \mathbb{E}(V)}{\partial e_L} = (1 - \eta)(\theta_L R - ce_L^{DN}) - \eta ce_L \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] = 0,$$

which imply that

$$e_H^{DN} = e_H^* \quad \text{and} \quad e_L^{DN} = \frac{(1 - \eta)\theta_L R}{c \left[ 1 - \eta \left( \frac{\theta_L}{\theta_H} \right)^2 \right]} < e_L^*. $$

It is noteworthy that, in the presence of asymmetric information problem, shareholders’ value decreases in two aspects: first, shareholders induce under-effort in unfavorable state $\theta_L$ (i.e., the effort distortion does not occur in state $\theta_H$); second, the optimal compensation $\omega_H^{SB}$ and $\omega_L^{SB}$, determined by (4) and (5), respectively, pays information rent to the manager in state $\theta_H$ (i.e., the managerial pay net of effort cost is zero in state $\theta_L$).
3.3 Case of interest: unobservable $\theta$ with takeover bids

Having defined the agency problem, we extend our analysis to the case in which shareholders have an opportunity to receive a takeover bid from potential acquirers. As shown below, the possibility of synergistic takeover provides managerial disciplinary effect to shareholders.

We first consider the potential acquirer’s optimal bidding strategy given $P(\hat{\theta})$ which is specified in the managerial compensation contract as the minimum price at which shareholders commits to selling the firm. Recall that, in our setting, $P(\hat{\theta})$ is publicly observable and the acquirer maximizes the firm value after takeover (i.e., the first-best effort will be chosen after takeover). When the realized state is $\theta_i$ ($i = l, h$) and the manager reports $\hat{\theta_j}$ ($j = l, h$), the takeover is profitable for the acquirer, provided that

$$P(\hat{\theta_j}) \equiv p_j \leq \theta_i e_i^* R - \frac{1}{2} ce_i^* + \pi \iff \pi \geq p_j - (\theta_i e_i^* R - \frac{1}{2} ce_i^*),$$

(10)

where $e_i^*$ is the first-best effort level in $\theta_i$-state. The condition (10) is intuitive: the takeover is more profitable for the acquirer as the synergy $\pi$ is higher or as the required takeover premium $p_j$ is lower relative to the target firm’s first-best net cash flow, net of the effort cost, $\theta_i e_i^* R - \frac{1}{2} ce_i^*$.

Now we find the optimal contract by solving the shareholders’ optimization problem at $t = 0$. We define $\pi_{ij}$ as the minimum synergy value that initiates takeover, i.e., $\pi_{ij} \equiv p_j - (\theta_i e_i^* R - \frac{1}{2} ce_i^*)$ from (10). Then, at $t = 0$, shareholders and the manager anticipate that takeover will occur with probability $1 - H(\pi_{ij})$. To facilitate the presentation, we abbreviate the probability of “no takeover” $H(\pi_{ij})$ as $H_{ij}$. The optimal contract solves the following problem:

$$\max_{\langle e_i, \omega_i, p_i \rangle} \mathbb{E}(V) = \eta[H_{HH}(\theta_H e_R - \omega_H) + (1 - H_{HH})p_H] + (1 - \eta)[H_{LL}(\theta_L e_R - \omega_L) + (1 - H_{LL})p_L],$$
subject to

\[ \omega_L - \frac{1}{2} ce_L^2 \geq 0, \quad \text{(PCL)} \]
\[ \omega_H - \frac{1}{2} ce_H^2 \geq 0, \quad \text{(PCH)} \]
\[ H_{LL}(\omega_L - \frac{1}{2} ce_L^2) \geq H_{LH}(\omega_H - \frac{1}{2} ce^2_H), \quad \text{(ICL)} \]
\[ H_{HH}(\omega_H - \frac{1}{2} ce_H^2) \geq H_{HL}(\omega_L - \frac{1}{2} ce^2_L), \quad \text{(ICH)} \]

where \( \bar{e} = \frac{\theta_H}{\theta_L} e_H \) and \( e = \frac{\theta_L}{\theta_H} e_L. \)\(^{16}\)

As in the standard adverse selection models, (PCL) and (ICH) are binding constraints, implying the following first order conditions:

\[ \frac{\partial \mathbb{E}(V)}{\partial e_H} = \eta(\theta_H R - ce_H) = 0, \]
\[ \frac{\partial \mathbb{E}(V)}{\partial e_L} = (1 - \eta)H_{LL}(\theta_L R - ce_L) - \eta H_{HL} ce_L \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] = 0, \]
\[ \frac{\partial \mathbb{E}(V)}{\partial p_H} = \eta[h_{HH}(\theta_H e_H R - \frac{1}{2} ce^2_H) + (1 - H_{HH}) - h_{HH} p_H] = 0, \]
\[ \frac{\partial \mathbb{E}(V)}{\partial p_L} = (1 - \eta)[h_{LL}(\theta_L e_L R - \frac{1}{2} ce^2_L) + (1 - H_{LL}) - h_{LL} p_L] - \eta h_{HL} \left[ \frac{1}{2} c(e^2_L - \epsilon^2) \right] = 0. \]

From the first order conditions, we find that the takeover prices are determined as follows:

**Lemma 1** When dispersed shareholders do not observe the realized state of industry \( \theta_i \) \((i = h, l)\), the takeover prices \( p^T_D H \) and \( p^T_D L \) are determined at

\[ p^T_D H = \frac{1}{2} \frac{(\theta_H R)^2}{c} + \frac{1 - H_{HH}}{h_{HH}}, \]

\[ p^T_D L = \frac{1}{2} \frac{(\theta_L R)^2}{c} + \frac{1 - H_{HL}}{h_{HL}}. \]

\(^{16}\)Although not unique, it is optimal to set the managerial compensation to 0 in the event of a takeover. To see this, let \( \omega^T \) represent the compensation in the event of a takeover. ICH becomes:

\[ H_{HH}(\omega_H - \frac{1}{2} ce_H^2) + (1 - H_{HH}) \omega^T_H \geq H_{HL}(\omega_L - \frac{1}{2} ce_L^2) + (1 - H_{HL}) \omega^T_L, \quad \text{(ICH)} \]

which implies that \( \omega^T_L \) can be reduced without violating the participation constraint, while reducing the wage cost and relaxing the incentive compatibility. Likewise, \( \omega^T_H \) can be reduced to improve profitability without violating the participation constraint.
\[ p_{DL}^{DT} = \left( \frac{\theta_L R}{ca} \right)^2 \left( 1 - \frac{1}{2a} \right) + \frac{1 - H_{LL}}{h_{LL}} - \frac{\eta}{1 - \eta} h_{HL} \left( \frac{\theta_L R}{\theta_H} \right)^2 \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right], \]

where \( a = 1 + \frac{\eta}{1 - \eta} \frac{H_{HL}}{h_{LL}} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] > 1. \)

Notice that, relative to the case in which shareholders observe \( \theta \), the takeover price in favorable state \( p_{DL}^{DT} \) does not change but the price for unfavorable state \( p_{DL}^{DT} \) becomes lower.\(^{17}\)

The first terms in both \( p_{DL}^{DT} \) and \( p_{DL}^{DT} \) correspond to the net cash flow value, net of effort cost. The second term, \( \hat{\pi} \equiv \frac{1 - H_{ii}}{h_{ii}} \) is the critical value of business synergy \( \pi \) at which the acquirer breaks even when \( \theta \) is publicly observable. The third term in \( p_{DL}^{DT} \), which does not show up when \( \theta \) is observable, represents the reduction in the takeover price proportionate to the information rent foregone to the manager.

Now consider how the possibility of synergistic takeover affects the managerial agency problem:

**Proposition 1** The probability of the takeovers depends on the realized state as well as the managerial report. Specifically, \( H_{HH} \geq H_{LL} \geq H_{HL}. \)

Proposition 1 states that the probability of takeover (i.e., \( 1 - H_{ij} \) for \( i, j \in \{L, H\} \)) is higher when the manager underreports the state than when the manager truthfully reports. Conditional on \( \theta_L \)-state being declared, the takeover is more profitable for the acquirer if the manager misreported, because the acquirer knows it can generate the first best outcome of \( \theta_H \)-state. Put it another way, incumbent shareholders undervalue the firm due to managerial misreport and, thus, ask low takeover premium which, in turn, makes the deal more profitable for the acquirer. This feature of takeover is referred to as a managerial disciplining device by Scharfstein (1988). To utilize the disciplinary effect, shareholders must set the takeover price to be lower when the manager reports \( \hat{\theta}_L \) than when she reports \( \hat{\theta}_H \). This implies that the synergistic takeover is more likely to occur in unfavorable state even when the manager truthfully reports. Proposition 1 also implies that the critical synergy values are

\(^{17}\)The takeover price when \( \theta \) is publicly observable is derived in the Appendix.
characterized as $\pi_{HH} \geq \pi_{LL} \geq \pi_{HL}$, and, furthermore, $\pi_{LL}$ can be negative, i.e., the takeover price for unfavorable state is set low enough even for the acquire with negative synergy find the deal profitable.

Finally, we characterize the optimal managerial wage scheme $(\omega_{DT}^{H}, \omega_{DT}^{L})$ and the corresponding effort choices $(e_{DT}^{H}, e_{DT}^{L})$:

\[
\omega_{DT}^{H} = \frac{(\theta_{H}R)^2}{2c} + \frac{H_{HL}(\theta_{L}R)^2}{2ca^2} \left[ 1 - \left( \frac{\theta_{L}}{\theta_{H}} \right)^2 \right],
\]
\[
\omega_{DT}^{L} = \frac{(\theta_{L}R)^2}{2ca^2},
\]
\[
e_{DT}^{H} = \frac{\theta_{H}R}{c},
\]
\[
e_{DT}^{L} = \frac{\theta_{L}R}{c \left[ 1 + \frac{\eta}{1-\eta} \frac{H_{HL}}{H_{HH}} \left( 1 - \left( \frac{\theta_{L}}{\theta_{H}} \right)^2 \right) \right]}.
\]

As in the case without takeover bids, the manager receives information rent in favorable state while taking zero surplus in unfavorable state. The second term in $\omega_{DT}^{H}$ measures the manager’s information rent which is proportionate to $H_{HL}$. If the manager misreports, the firm is more likely to be taken over, i.e., $H_{HL} \leq H_{HH}$ or $H_{HL} / H_{HH} < 1$. This implies that shareholders use the synergistic takeover to discipline the manager and, in doing so, they reduce the information rent. This managerial disciplinary effect strictly improves the efficiency relative to the case without the takeover. The efficiency gains can be also confirmed in the optimal effort choice $e_{DT}^{L}$ which is higher than $e_{DN}^{L}$ which is the optimal effort choice when there is no takeover market.

4 Concentrated ownership firms

In this section, we analyze the concentrated ownership firms that have controlling shareholders. We first analyze the baseline model where controlling shareholders face the same managerial agency problem with dispersed shareholders, and then we extend the model by
consider the monitoring technology of the controlling shareholders.

4.1 Baseline case: without monitoring

In this setting, controlling shareholders, e.g., family owners, receive non-transferable private benefits of control $B$ that increases the takeover premium required by the controlling shareholder to sell out the firm. The higher takeover premium certainly lowers the probability of takeover, but as formally shown below, it does not necessarily reduce the managerial disciplinary effect of synergistic takeover.

Consider the controlling shareholders’ optimization problem at $t = 0$. For the ease of exposition, we do not change the notations used in Section 3. Then, controlling shareholders’ problem can be written as:

$$\max_{(e_i, \omega_i, p_i)} \mathbb{E}(V) = \eta [H_{HH}(\theta_H e_H R - \omega_H + B) + (1 - H_{HH}) p_H] + (1 - \eta) [H_{LL}(\theta_L e_L R - \omega_L + B) + (1 - H_{LL}) p_L],$$

subject to

$$\omega_L - \frac{1}{2} c e_L^2 \geq 0, \quad \text{(PCLCN)}$$
$$\omega_H - \frac{1}{2} c e_H^2 \geq 0, \quad \text{(PCHCN)}$$
$$H_{LL}(\omega_L - \frac{1}{2} c e_L^2) \geq H_{LH}(\omega_H - \frac{1}{2} c e_H^2), \quad \text{(ICLCN)}$$
$$H_{HH}(\omega_H - \frac{1}{2} c e_H^2) \geq H_{HL}(\omega_L - \frac{1}{2} c e_L^2). \quad \text{(ICHCN)}$$

Relative to the dispersed ownership case, there is an addition term of private benefits $B$ in controlling shareholders’ objective function. The first order conditions can be easily derived from those in dispersed ownership case and, thus, we place them in the Appendix. In what follows, we present the key results by comparing them with the dispersed ownership case.

Proposition 2 formalizes how the presence of controlling shareholders affects the incidence
of takeover:

**Proposition 2** Concentrated ownership firms are sold at strictly higher takeover price than dispersed ownership firms.

As noted previously, controlling shareholders require higher takeover premium as a reward for foregoing the private benefits of control $B$ and, thus, make the takeover deal less profitable for acquirers than the deal with dispersed ownership firms. More specifically, at the managerial contracting stage, the controlling shareholders set the takeover price high enough to fully compensate for $B$:

**Corollary 1** Controlling shareholders set the takeover prices to be fully compensated for the private benefit control $B$.

Now we turn to the managerial disciplinary effect of synergistic takeover for concentrated ownership firms. While the controlling shareholders reduce the likelihood of takeover, they do not necessarily diminish the managerial disciplinary effect of synergistic takeover, and more specifically, their effect on the disciplinary mechanism relies on the firm’s growth options, i.e., the business synergies with potential acquirers. In this setting, the firm’s growth option is modeled as $H(\cdot)$ which is the distribution function of synergy value $\pi$. More specifically, for the purpose of our analysis that compares two ownership structure, we focus on the subinterval of $\pi$ in which the critical synergy values $(\pi_{HH}, \pi_{LL}, \pi_{HL})$ are shifted due to the private benefit $B$ (refer to Corollary 1). Proposition 3 characterizes how the firm’s growth option affects the relation between ownership concentration and the market disciplinary mechanism:

**Proposition 3** For $\theta = \theta_i$ and $\hat{\theta} = \hat{\theta}_j$ ($i,j \in \{L,H\}$, let $\pi_{ij}^C$ and $\pi_{ij}^D$ denote the critical synergy value under concentrated ownership and dispersed ownership, respectively, and $H_{ij}^C$ and $H_{ij}^D$ denote the corresponding probabilities of “no takeover.”

1. If $H(\cdot)$ is log-convex, the disciplinary effect of takeovers is stronger in firms with concentrated shareholders, i.e., $\frac{H_{ij}^C}{H_{ij}^L} \leq \frac{H_{ij}^D}{H_{ij}^L}$ and $\frac{H_{HH}^C}{H_{HH}^L} \leq \frac{H_{HH}^D}{H_{HH}^L}$.
2. If $H(\cdot)$ is log-concave, the disciplinary effect of takeovers is weaker in firms with concentrated shareholders, \(\frac{H_{HL}}{H_{LL}} > \frac{H_{HL}}{H_{HH}}\) and \(\frac{H_{HL}}{H_{HH}} > \frac{H_{HL}}{H_{HH}}\).

The log-convex case in Proposition 3 may seem counter-intuitive at the first glance: all else being equal, the control premium reinforces the effectiveness of the market disciplinary mechanism even though it reduces the likelihood of takeover incidence. To discuss the intuition behind this result, first consider two competing effect of private benefit of controlling shareholders on the market disciplinary mechanism: on one hand, it makes the controlling shareholders less likely to sell out and, thus, entrenches the manager from takeover threat even when she underreports the states (i.e., \(H_{HL}^C > H_{HH}^D\)); on the other hand, it also reduces the probability of takeover when the manager truthfully reports favorable state (i.e., \(H_{HH}^C > H_{HH}^D\)). Controlling shareholders’ commitment to selling the firm at higher price when favorable state is reported increases the manager’s opportunity cost of underreporting the state. Therefore, the presence of controlling shareholders strengthens the managerial disciplinary effect of synergistic takeover if the latter dominate, or it weakens the disciplinary effect, otherwise.

Now consider how the log-convexity of $H(\cdot)$ is associated with these two competing effects, respectively. The log-convexity implies that the increase in takeover price exponentially reduces the probability of takeover incidence. Recall that the critical synergy value that makes the acquire break even is higher when the manager truthfully reports the favorable state than when she underreports it. Thus, given that $H(\cdot)$ is log-convex, the increase in takeover price associated with the control benefit reduces the probability of takeover more when the manager truthfully reports, i.e., the latter effect dominates the former. Formally, under the log-convexity of $H(\cdot)$, we find that \(\frac{H_{HL}^C}{H_{HH}^H} \leq \frac{H_{HL}^D}{H_{HH}^H}\), which, in turn, relaxes the truth-telling incentive compatibility constraint.

Intuitively, the firms with log-convex distribution function of synergy values can be interpreted as high growth firms, since, for those firms, acquirers are more likely to find higher takeover synergies. Given the high potential synergy value, the truthful managerial report
and the subsequent increase in takeover premium themselves do not significantly deter the
takeover initiated by those who seek the business synergies from acquiring the firm. The
takeover deterrence by truthful report however can be complemented by the control premium
required by controlling shareholders. That is, when the controlling shareholders require ad-
ditional premium, the profitability of takeover becomes more sensitive to the current value of
target firm, determined by the managerial report. This intuition also explains why controlling
shareholders negatively affect the market disciplinary mechanism when $H(\cdot)$ is log-concave.

In what follows, we will focus on the log-convexity case for ease of exposition, but the results
will be flipped in log-concave case.

Proposition 4 reports how ownership structure affects the managerial compensation struc-
ture and consequently the managerial effort choice:

**Proposition 4** For $\theta = \theta_i$ ($i = h, l$), let $\omega_i^C$ and $\omega_i^D$ denote the optimal managerial wage
scheme under concentrated and dispersed ownership, respectively, and $e_i^C$ and $e_i^D$ denote the
corresponding managerial effort choices. If $H(\cdot)$ is log-convex, then:

1. $\omega_L^C > \omega_L^D$ and $\omega_H^C - \omega_L^C < \omega_H^D - \omega_L^D$,
2. $e_L^C > e_L^D$ and $e_H^C = e_H^D$.

As shown in proposition 2, the market disciplinary mechanisms and the managerial performance-
based pay are substitutes. In the log-convex synergy distribution case, the controlling share-
holders strengthens the market disciplinary mechanism, and therefore they reduce the use of
performance-based pay. The efficiency gains from ownership concentration in these firms is
also confirmed in the managerial effort choice for the unfavorable state. Relative to dispersed
shareholders, controlling shareholders utilize the market disciplinary mechanism, and pro-
vide the manager reporting unfavorable state with larger wage which in turn induces higher
effort.

Formally, the optimal wage scheme for the manager who reports favorable state is deter-
mined as:

\[ \omega_H = \frac{(\theta_H R)^2}{2c} + \frac{H_{HL}}{H_{HH}} \frac{(\theta_L R)^2}{2ca^2} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right], \]

where the second term corresponds to the information rent, which increases with the ratio \( \frac{H_{HL}}{H_{HH}} \). In the log-convex case, as shown in Proposition 2, the private benefit of control reduces this ratio, i.e., strengthens the market disciplinary mechanism. The reduced information rent in turn allows the controlling shareholders to offer large pay to the manager who report unfavorable state and induce higher effort, as confirmed in the following optimal managerial effort choice:

\[ e_C^L = \frac{\theta_L R}{c \left[ 1 + \frac{\eta}{1-\eta} \frac{H_{HL}}{H_{LL}} \left( 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right) \right]}, \]

which decreases with the ratio \( \frac{H_{HL}}{H_{LL}} \). Proposition 2 shows that, in the log-convex case, the private benefit of controlling shareholders reduces this ratio.

These results imply that, in the log-convex case, controlling shareholders can pay less information rent to the manager in favorable state, and induce higher managerial effort in unfavorable state. Thus, in this case, the presence of controlling shareholders can increase the operating efficiency of incumbent managers. It it is noteworthy that, notwithstanding the operating efficiency gains from ownership concentration, minor shareholders still prefer the dispersed ownership structure. Intuitively, controlling shareholders forego the profitable takeover bids for the sake of their control benefit, and this opportunity cost dominates the operating efficiency gains from concentrated ownership structure. In sum:

**Proposition 5** If \( H \) is log-convex, the incumbent manager shows better operating performance in concentrated ownership firms than in dispersed ownership while the market value is higher under the dispersed ownership structure.
4.2 Extension: monitoring by Controlling Shareholders

In this section, we extend the model by considering the managerial monitoring function of controlling shareholders.\(^{18}\) We model the monitoring technology as follows: after the takeover stage (i.e., \(t = 2\)), controlling shareholders can observe the realized state of industry \(\theta\) with probability \(\lambda\) by incurring a private cost \(\frac{1}{2}\kappa\lambda^2\). If controlling shareholders observe \(\theta\) from monitoring, they do not use the managerial report \(\hat{\theta}\) and instead enforce the first-best effort choice without foregoing surplus (i.e., they pay the effort cost only).\(^{19}\)

Given that the managerial agency problem cannot be fully eliminated by the takeover threat and managerial compensation (i.e., \(H_{HL} > 0\) when controlling shareholders do not monitor), the monitoring function of controlling shareholders can complement the other two governance mechanisms in our setting. Without loss of generality, we focus on the case in which controlling shareholders monitor only when the manager reports \(\hat{\theta} = \hat{\theta}_L\), since the manager does not have an incentive to misreport in unfavorable state (i.e., \(\hat{\theta}_H\) is always truthful).

Now consider the optimization problem of controlling shareholders who can monitor \(\theta\):

\[
\max_{\langle \varepsilon, \omega, p, \lambda \rangle} \eta [H_{HH}(V^S_H - \omega_H + B) + (1 - H_{HH})p_H] + (1 - \eta) [H_{LL}(V^S_L - \omega_L + B - \frac{1}{2}\kappa\lambda^2) + (1 - H_{LL})p_L],
\]
subject to

\[
\omega_L - \frac{1}{2} \varepsilon_H^2 \geq 0, \quad (PCL_{CM})
\]
\[
\omega_H - \frac{1}{2} \varepsilon_H^2 \geq 0, \quad (PCH_{CM})
\]
\[
H_{LL}(\omega_L - \frac{1}{2} \varepsilon_L^2) \geq H_{LL}(\omega_H - \frac{1}{2} \varepsilon_H^2), \quad (ICL_{CM})
\]
\[
H_{HH}(\omega_H - \frac{1}{2} \varepsilon_H^2) \geq H_{HH}(1 - \lambda)(\omega_L - \frac{1}{2} \varepsilon_L^2), \quad (ICH_{CM})
\]

\(^{18}\)Relative to dispersed shareholders, controlling shareholders can monitor the manager better due to, e.g., the absence of free-riding problem, the required knowledge, and fixed monitoring costs.

\(^{19}\)While this modeling choice simplifies our analysis, one may question the severity of punishment after monitoring the managerial misreport in practice. In our setting, more severe punishment relaxes the truth-telling incentive compatibility constraint, but it does not change our main results qualitatively.
In our setting, market disciplinary mechanism of synergistic takeover is imperfect due to the possibility that the acquirer finds negative synergy value. That is, even though the potential acquirer perfectly observes the realized state $\theta$, it may not acquire the undervalued target firm in which the manager underreports the state. Controlling shareholders’ monitoring function, though it is costly and imperfect, can complement the market disciplinary mechanism, since, the probability of monitoring success $\lambda$ is independent of the distribution of synergy value $H(\cdot)$. Formally:

**Proposition 6** Concentrated shareholders monitor with a strictly positive intensity.

In Sections 3 and 4, we show that market disciplinary mechanism and the performance-based managerial pay are substitute governance mechanisms. Now we turn to the question how the internal monitoring interacts with these two governance mechanisms. It turns out that, while managerial compensation is substitute to both mechanisms, the interaction between depends on the distribution of synergies $H$. To analyse the interactions, we will focus on the relationship between the takeover mechanism and internal monitoring:

**Proposition 7** Internal monitoring and market disciplinary mechanisms are

1. complements if $H(\cdot)$ is log-convex,

2. substitutes if $H(\cdot)$ is log-concave.

Managerial performance-based pay is substitute for both mechanisms.

The intuition is as follows: to increase the effectiveness of the takeover mechanism as a disciplining device, the shareholders should lower the takeover price $p_L$. The lower price in turn reduces the critical synergy value $\pi_{HL}$ at which the acquirer breaks even, and, thus, it increases $1 - H_{HL}$ the probability of takeover conditional on the managerial underreport.
However, the commitment not to sell out as often if the manager is truthful, $H_{HH}$, remains unaffected by a decrease in $p_L$. Given the log-convexity of the distribution of synergies, the ratio $\frac{H_{HL}}{H_{HH}}$ decreases which in turn increases the effectiveness of the disciplinary effect. However, log-convexity also implies that lower $p_L$ increases the ratio $\frac{H_{HL}}{H_{LL}}$ increases (i.e., $H_{LL}$ decreases more than $H_{HL}$). From controlling shareholder’s perspective, the higher ratio lowers the efficiency of market disciplinary mechanism since they are willing to give out more firm value to the acquirer making the takeovers with negative synergies more likely to occur. Controlling shareholders can improve the outcome by increasing the monitoring intensity, conditional on the managerial report of unfavorable state $\hat{\theta}_L$. The same logic explains why the two mechanisms are substitutes if $H(\cdot)$ is log-concave. Finally, the substitutability between performance-based pay and other governance mechanisms naturally arises from the fact that performance-based pay is positively associated with the information rent, which is reduced by other mechanisms, foregone to the manager in favorable state.

Our results provide some key policy implications regarding the regulation on ownership concentration. In many countries, there are debates about the social cost of concentrated ownership structure, and some regulators (e.g., the European Commission) considers breaking up the concentrated ownership structure to facilitate the market for corporate control. Our analysis shows that the managerial disciplinary mechanism of synergistic takeover can be strengthened by the presence of controlling shareholders. While the control premium required by the controlling shareholders reduce the incidence of takeover even with positive synergy values, the internal managerial monitoring performed by these shareholders can complement the market disciplinary mechanism in high growth firms. Therefore, it is ambiguous whether dismantling concentrated ownership structure, in particular, in high growth firms would increase the firm value. Proposition 7 suggests that, if high growth firms are already owned by controlling shareholders, regulators may consider a policy that facilitates the market for corporate control in industry downturns to take advantage of internal monitoring.
5 Conclusion

In this paper we show the interaction between internal and external governance mechanisms in firms with controlling shareholders when agency problems arise due to information asymmetry. Contrary to common prior, disciplinary effect of synergistic takeovers can be stronger in smaller and growing firms with concentrated shareholders due to improvements in incentives for managerial self-selection. Specifically, the control premium provides the manager to deter the takeover threat by increasing the current value of the firm. In this case, the managerial entrenchment is consistent with improvements in shareholder value. The disciplinary effect acts as a complement to internal monitoring efforts of controlling shareholders in reducing the amount of incentives pay required to induce truthfulness. In contrast, the control premium in mature firms with few synergies isolates the manager from the takeover threat making incentive provision reliant on internal monitoring. In order to reduce the amount of ”distortion at the bottom”, controlling shareholders over-monitor the manager to ensure perpetuation of control.

The disciplining effect of synergistic takeovers is not without its costs. Incentive provision requires that shareholders accept relatively low bidding prices, even if it implies allowing takeovers with negative synergies. Furthermore, tailoring correct incentive pay requires a relatively high distortion to effort levels in the low states of the world. While controlling ownership is able to mitigate these concerns, the existence of control premium reduces the incidence of socially desirable synergistic improvements in the firm value.
Appendix

A.1 Shareholders’ optimal choice of takeover price when \( \theta \) is observable

The optimal contract under the first-best with takeovers, i.e., when the state is publicly observable/verifiable, solves the following program for the dispersed firm:

\[
\max_{(e_i, \omega_i, p_i)} \mathbb{E}(V) = \eta [H_{HH}(V_{H}^S - \omega_H) + (1 - H_{HH})p_H] + (1 - \eta)[H_{LL}(V_{L}^S - \omega_L) + (1 - H_{LL})p_L],
\]

subject to

\[
\omega_L - \frac{1}{2} c e_L^2 \geq 0, \quad \text{(PCL)}
\]

\[
\omega_H - \frac{1}{2} c e_H^2 \geq 0, \quad \text{(PCH)}
\]

In equilibrium participation constraints bind, and the optimal solutions satisfy

\[
e_i^* = \frac{\theta_i R}{c},
\]

\[
p_i^* = \frac{1}{2} \left( \frac{\theta_i R}{c} \right)^2 + \frac{1 - H_{ii}}{h_{ii}},
\]

\[
\omega_i^* = \frac{(\theta_i R)^2}{2c},
\]

for \( i \in (L, H) \). Hence, the pay for performance sensitivity is given by:

\[
\Delta \omega = \omega_H - \omega_L = \frac{R^2}{2c} (\theta_H^2 - \theta_L^2) = \frac{(\theta_H R)^2}{2c} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right].
\]
For the concentrated firm, the only difference is that $p_i^*$ will reflect an increase in $B$.

### A.2 Proof of Lemmas and Propositions

**Proof of Lemma 1** Omitted since it is straightforward from the first order conditions.

**Proof of Proposition 1**

We distinguish between three cases based on managerial report. To prove the proposition, we focus on critical values of $\pi_{ij} = p_j - Y_i^*$, where $Y_i^* \equiv \theta_i e_i^* R - \frac{1}{2} c e_i^2$ is the first-best value of net cash flow, net of the effort cost.

If the manager reports truthfully in the high state from the first order condition for $p_H$,

$$\pi_{HH} = p_H^* - Y_H^* = \frac{1 - H_{HH}}{h_{HH}} > 0,$$

which only depends on the distribution of $\pi$ and is the same as in the case of first best.

If the manager reports truthfully in the low state we have

$$\pi_{LL} = \hat{p}_L - Y_L^* = \frac{1 - H_{LL}}{h_{LL}} - \frac{\eta}{1 - \eta} \frac{h_{HL} \theta L R}{2ca^2} \left[ 1 - \left( \frac{\theta L}{\theta H} \right)^2 \right].$$

The second term in $H_{LL}$ reduces the critical $\pi$ at which takeovers occur compared to the first best with takeovers. Note that, under the first best inverse of the hazard rate is the same if the manager reports truthfully irrespective of the state of the world. From the properties of Cumulative Distribution Functions, we know that $H_{LL}$ must be non-decreasing in its arguments, and by continuity we have $H_{HH} > H_{LL}$.

If the manager misreports in the high state we have

$$\pi_{HL} = \hat{p}_L - Y_H^* = \frac{1 - H_{LL}}{h_{LL}} - \frac{\eta}{1 - \eta} \frac{h_{HL} (\theta L R)^2}{2ca^2} \left[ 1 - \left( \frac{\theta L}{\theta H} \right)^2 \right] + \frac{(\theta L R)^2}{ca} \left( 1 - \frac{1}{2a} \right) - \frac{1}{2} \left( \frac{\theta H R^2}{c} \right).$$

The first two terms in the $\pi_{HL}$ are the same as in $\pi_{LL}$, while the sum of the last two terms
is negative because it represents the difference between values net of cost under the second best and the first best respectively. Since the shareholders do not know whether the true state is low, they distort the takeover price conditional on low managerial report lower. If the true state is indeed high, this makes it more profitable for the acquirer which equates to higher probability of the takeover if the manager misreports. Comparing the quantities, \( \pi_{HH} \geq \pi_{LL} \geq \pi_{HL} \). Since \( H(\pi_{ij}) \) is non-decreasing function of \( \pi \) which is assumed to be continuous on the support \([\bar{\pi}, \bar{\pi}]\), implies \( H_{HH} \geq H_{LL} \geq H_{HL} \). Q.E.D.

**Proof of Proposition ??**

Pay for performance sensitivity implied by the optimal wage scheme \((\omega_H^{DT}, \omega_L^{DT})\) is

\[
\Delta \omega^{DT} = \omega_H^{DT} - \omega_L^{DT} = \frac{\theta_H R}{2c} + \frac{H_{HL}}{H_{HH}} \frac{\theta_L R}{2ca^2} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] - \frac{\theta_L R}{2ca^2} \\
= \frac{\theta_H R}{2c} - \frac{\theta_L R}{2ca^2} \left\{ 1 - \frac{H_{HL}}{H_{HH}} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] \right\}.
\]

If takeovers had no disciplining effect, i.e., setting \( H_{HL} = H_{LL} \) and \( H_{HL} = H_{HH} \) or equivalently setting the ratios \( \frac{H_{HL}}{H_{LL}} = \frac{H_{HL}}{H_{HH}} = 1 \), we get

\[
\Delta \omega^{DN} = \omega_H^{DN} - \omega_L^{DN} = \frac{\theta_H R}{2c} - \frac{(1 - \eta)(\theta_L R)^2}{2c} \left[ 1 - \eta \left( \frac{\theta_L}{\theta_H} \right)^2 \right]^2.
\]

which is the same as if we subtract the equation 4 from the equation 5 and substitute \( e_H^{DN} \) and \( e_L^{DN} \). Note that the first term is the same in both cases. However, the second term is larger in \( \Delta \omega^{DT} \) than \( \Delta \omega^{DN} \) if \( \frac{H_{HL}}{H_{LL}} < 1 \) and \( \frac{H_{HL}}{H_{HH}} < 1 \), since \( e_L^{DT} > e_L^{DN} \) and \( \left\{ 1 - \frac{H_{HL}}{H_{HH}} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] \right\} > 1 - 1 \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] = \left( \frac{\theta_L}{\theta_H} \right)^2 \) implying \( \Delta \omega^{DN} > \Delta \omega^{DT} \). Q.E.D.

**Proof of Proposition 2**

By the usual arguments, PCL and ICH are binding. The first order conditions for the program are
\begin{align*}
\frac{\partial E(V)}{\partial e_H} &= \eta(\theta_H R - ce_H) = 0, \\
\frac{\partial E(V)}{\partial e_L} &= (1 - \eta)H_{LL}(\theta_L R - ce_L) - \eta H_{HL} ce_L \left[1 - \left(\frac{\theta_L}{\theta_H}\right)^2\right] = 0, \\
\frac{\partial E(V)}{\partial p_H} &= \eta[h_{HH}(\theta_H e_H R - \frac{1}{2}ce_H^2 + B) + (1 - H_{HH}) - h_{HH}p_H] = 0, \\
\frac{\partial E(V)}{\partial p_L} &= (1 - \eta)[h_{LL}(\theta_L e_L R - \frac{1}{2}ce_L^2 + B) + (1 - H_{LL}) - h_{LL}p_L] - \eta h_{HL} \left[\frac{1}{2}c(e_L^2 - e^2)\right] = 0.
\end{align*}

Further, the second order conditions are:

\begin{align*}
\frac{\partial^2 E(V)}{\partial e_H^2} &= -\eta H_{HHC}, \\
\frac{\partial^2 E(V)}{\partial e_L^2} &= (1 - \eta)H_{LL} - \eta H_{HLC} \left[1 - \left(\frac{\theta_L}{\theta_H}\right)^2\right], \\
\frac{\partial^2 E(V)}{\partial p_H^2} &= -\eta H_{HH} \left(2h_{HH} + h'_{HH} \frac{1 - H_{HH}}{h_{HH}}\right), \\
\frac{\partial^2 E(V)}{\partial p_L^2} &= -(1 - \eta) \left\{2h_{LL} + h'_{LL} \frac{1 - H_{LL}}{h_{LL}} - h'_{LL} \frac{\eta}{h_{LL}} \frac{h_{HL}(\theta_L R)^2}{2ca^2} \left[1 - \left(\frac{\theta_L}{\theta_H}\right)^2\right]\right\}, \\
\frac{\partial^2 E(V)}{\partial e_H \partial p_H} &= \eta h_{HH} \left(\theta_H R - \frac{\theta_H R}{c}\right),
\end{align*}
\[
\frac{\partial^2 E(V)}{\partial e_L \partial p_L} = (1 - \eta)h_{LL}(\theta_L R - \frac{\theta_L R}{a}) - \eta h_{HL} \frac{\theta_L R}{a} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right],
\]
with the rest of unique cross-partials are zero. Hence, the determinant of the Hessian matrix, \(\Delta\), is given as
\[
\Delta = \begin{vmatrix}
\frac{\partial^2 E(V)}{\partial e_H^2} & 0 & 0 & 0 \\
0 & \frac{\partial^2 E(V)}{\partial e_L^2} & 0 & \frac{\partial^2 E(V)}{\partial e_L \partial p_L} \\
\frac{\partial^2 E(V)}{\partial e_H \partial p_H} & 0 & \frac{\partial^2 E(V)}{\partial p_L^2} & 0 \\
0 & \frac{\partial^2 E(V)}{\partial p_L \partial e_L} & 0 & \frac{\partial^2 E(V)}{\partial p_L^2}
\end{vmatrix} < 0
\]
if \(\frac{\partial^2 E(V)}{\partial e_L^2} \frac{\partial^2 E(V)}{\partial p_L^2} - \left\{ \frac{\partial^2 E(V)}{\partial p_L \partial e_L} \right\}^2 < 0\) which we assume to hold. Next, from the first order condition we derive the expressions for \(\hat{p}_C^H\) and \(\hat{p}_C^L\), which relative to the dispersed firm reflect the private benefits \(B\)
\[
\hat{p}_C^H = \frac{1}{2} \left( \frac{\theta_H R}{c} \right)^2 + \frac{1 - H_{HH}}{h_{HH}} + B > \hat{p}_D^H,
\]
\[
\hat{p}_C^L = \frac{(\theta_L R)^2}{ca} \left( 1 - \frac{1}{2a} \right) + \frac{1 - H_{LL}}{h_{LL}} + B - \eta \frac{h_{HL} (\theta_L R)^2}{1 - \eta h_{LL}} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] > \hat{p}_D^L.
\]
Hence, the critical values of synergies for which the takeovers occur are
\[
\pi_{HH}^C = \hat{p}_H^* - Y_{H}^* = \frac{1 - H_{HH}}{h_{HH}} + B > \pi_{HH}^D,
\]
which only depends on the distribution of \(\pi\) and is the same as in the case of first best.
If the manager reports truthfully in the low state we have
\[
\pi_{LL}^C = \hat{p}_L^* - Y_{L}^* = \frac{1 - H_{LL}}{h_{LL}} + B - \frac{\eta}{1 - \eta} \frac{h_{HL} \theta_L R}{2ca^2} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] > \pi_{LL}^D.
\]
The second term in \(H_{LL}\) reduces the critical \(\pi\) at which takeovers occur compared to the
first best with takeovers. Note that, under the first best inverse of the hazard rate is the same if the manager reports truthfully irrespective of the state of the world. From the properties of Cumulative Distribution Functions, we know that $H_{LL}$ must be non-decreasing in its arguments, and by continuity we have $H_{HH} > H_{LL}$.

If the manager misreports in the high state we have

$$
\pi_{HLL}^{C} = \hat{p}_L - Y_H^* = \frac{1 - H_{LL}}{h_{LL}} + B - \frac{\eta}{1 - \eta} \frac{h_{HL}(\theta_L R)^2}{2ca^2} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] + \frac{(\theta_L R)^2}{ca} \left( 1 - \frac{1}{2a} \right) \frac{1}{c} (\theta_H R)^2 > \pi_{HLL}^{D},
$$

Since $\frac{\partial H_{ij}}{\partial \pi_{ij}} > 0$ and $\frac{\partial \pi_{ij}}{\partial p_j} > 0$, $H_{ij}^C > H_{ij}^D$ and $H_{ii}^C > H_{ii}^D$ for $i, j \in (L, H)$ and $i \neq j$. Hence, a shifter $B$ (being the only additional term) increases the takeover price that concentrated shareholders demand and the critical level of synergies for which the takeover occurs, $\pi_{ij}$, and in turn increasing all the $H_{ij}$ relative to the dispersed case. Q.E.D.

**Proof of Proposition 3**

In Proposition 1, we have established that the takeover prices and critical values of synergies for concentrated firm will reflect the additional term $B$ which shifts the $H(\pi_{ij})$ function.

Assume that the shift in $B$ occurs in the relevant range. Since $H(\cdot)$ is assumed to be continuous and twice differentiable, it remains to show how does the ratio $\frac{H_{ij}^C}{\pi_{ij}}$, for $i, j \in (L, H)$ and $i \neq j$, change with an increase in $B$. To prove the first part we must show that the ratios are increasing in $B$ if the distribution is log-convex in the relevant range. Since $H_{HL}$ and $H_{LL}$ are both functions of $p_L$, taking the derivative of the ratio yields

$$
\frac{\partial (H_{HL}/H_{LL})}{\partial B} = \frac{h_{HL}H_{LL} - h_{LL}H_{HL}}{H_{LL}^2} \leq 0,
$$

if the numerator is negative. Rearranging yields $h_{HL}H_{LL} \leq h_{LL}H_{HL}$ or

$^{A.1}$Note that the difference between $p_j$ and $\pi_{ij}$ is the optimal value of the firm under the bidder, $Y_i^*$, which is independent of the manager/controlling shareholder choices.
where \( \frac{h_{HL}}{H_{HL}} \) is the reversed hazard ratio. By Lemma 1, \( H_{HH} \geq H_{LL} \geq H_{HL} \), and by Proposition 1, \( H_{HL}^C > H_{HL}^P \) and \( H_{LL}^C > H_{LL}^P \), and since the distribution is monotonic and increasing, the condition implies increasing reverse hazard rate (IRHR), i.e., \( \frac{\partial h}{\partial p_L} > 0 \), or that the log of distribution function \( H(\hat{\pi}) \) is convex (see Navarro and Shaked (2010)). Since by definition \( \frac{\partial h}{\partial B} > 0 \) and by Proposition 1, \( H_{HL}^C > H_{HL}^D \) and \( H_{LL}^C > H_{LL}^D \), this condition also must hold for a shifter \( B \).

Next, note that \( H_{HH} \) is a function of \( p_H \) and that \( p_L \neq p_H \). However, inspection of \( p_H^C \) and \( p_L^C \) indicates that \( B \) affects \( p_H \) by more since the inverse hazard ratio must be non-increasing if the problem is to be quasi-concave. Given that \( H_{HH} > H_{HL} \) and with log-convexity, it must be that \( \frac{\partial h_{HL}}{\partial B} > 0 \). By analogy, log-concavity follows trivially. Q.E.D.

**Proof of Proposition 4**

Given that \( H_{LL} \geq H_{HL} \) and \( H_{HH} \geq H_{HL} \) and the log-convexity of \( H \), by Proposition 2 we have \( \frac{H_{HL}^C}{H_{HL}^L} \leq \frac{H_{HL}^D}{H_{HL}^P} \) which implies (i). In order to prove (ii), let:

\[
\Delta \hat{\omega} = \hat{\omega}_H - \hat{\omega}_L = \frac{(\theta_H R)^2}{2c} + \frac{H_{HH}}{H_{HH}} \frac{(\theta_L R)^2}{2ca^2} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] - \frac{(\theta_L R)^2}{2ca^2} 
\]

\[
= \frac{(\theta_H R)^2}{2c} - \frac{(\theta_L R)^2}{2ca^2} \left\{ 1 - \frac{H_{HL}}{H_{HH}} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] \right\}.
\]

Since \( \frac{H_{HL}^C}{H_{HH}^H} < \frac{H_{HL}^D}{H_{HH}^H} \), and by part (i), the difference between the first and second term is smaller. Q.E.D.

**Proof of Proposition 5** From Proposition 4 part (i), \( \hat{V}_L^C > \hat{V}_L^D \) and \( \hat{V}_H^C = \hat{V}_H^D = \hat{V}_H^* \). However, this is a sufficient condition as we only require convexity over the relevant subinterval. For a list of distributions that satisfy the property of increasing reverse hazard rate over the entire interval, see Bagnoli and Bergstrom (2005).
Since, $\Delta \omega^C < \Delta \omega^D$ is needed to achieve $V_H^*$, and since for the range of firm values where $V_L^* > \hat{V}_L^C$, $V_i$ net of effort cost is concave in $e_i$ then $E_\theta(V^C) > E_\theta(V^D)$.

**Proof of Proposition 6**

The first order conditions are:

$$\frac{\partial E(V)}{\partial e_H} = \eta H_{HH}(\theta_H R - c e_H) = 0,$$

$$\frac{\partial E(V)}{\partial e_L} = (1 - \eta)H_{LL}(\theta_L R - c e_L) - \eta H_{HL}(1 - \lambda)c e_L \left[1 - \left(\frac{\theta_L}{\theta_H}\right)^2\right] = 0,$$

$$\frac{\partial E(V)}{\partial \lambda} = -(1 - \eta)H_{LL} \kappa \lambda + \eta H_{HL} \frac{1}{2} c e_L^2 \left[1 - \left(\frac{\theta_L}{\theta_H}\right)^2\right] = 0.$$

$$\frac{\partial E(V)}{\partial p_H} = \eta [h_{HH}(\theta_H e_H R - \frac{1}{2} c e_L^2 + B) + (1 - H_{HH}) - h_{HH} p_H] = 0,$$

$$\frac{\partial E(V)}{\partial p_L} = (1 - \eta) [h_{LL}(\theta_L e_L R - \frac{1}{2} c e_L^2 + B - \frac{1}{2} \kappa \lambda^2) + (1 - H_{LL}) - h_{LL} p_L] - \eta h_{HL}(1 - \lambda) \frac{1}{2} c e_L^2 \left[1 - \left(\frac{\theta_L}{\theta_H}\right)^2\right] = 0.$$

Assuming that the second order conditions hold, rearranging yields:

$$\lambda^* = \frac{\eta}{1 - \eta} \frac{H_{HL} c e_L^2}{2 \kappa} \left[1 - \left(\frac{\theta_L}{\theta_H}\right)^2\right],$$

(A.1)

$$\hat{e}_H = \frac{\theta_H R}{c},$$

which is the same as the first best. Next,
\[
\hat{e}_L = \frac{\theta_L R}{c \left[ 1 + \frac{\eta}{1 - \eta} \frac{H_{HL}}{h_{LL}} (1 - \lambda) \left( 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right) \right]},
\]

which is lower than the \( \hat{e}_L \) without the takeover market. The following equations represent the optimal prices at which the incumbent shareholders are willing to sell out to the raider:

\[
\hat{p}^F_H = \frac{1}{2} \frac{(\theta_H R)^2}{c} + B + \frac{1 - H_{HH}}{h_{HH}},
\]

\[
\hat{p}^F_L = \theta_L e_L R - \frac{1}{2} \frac{c e_L^2}{a} + B + \frac{1}{2} \kappa \lambda^2 + \frac{1 - H_{LL}}{h_{LL}} - \eta \frac{h_{HL}}{1 - \eta} h_{LL} (1 - \lambda) \frac{1}{2} \frac{c e_L^2}{a} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right]
\]

\[= \left( \frac{\theta_L R}{c a} \right)^2 \left( 1 - \frac{1}{2a} \right) + B + \frac{1}{2} \kappa \lambda^2 - \frac{1 - H_{LL}}{h_{LL}} - \eta \frac{h_{HL}}{1 - \eta} h_{LL} (1 - \lambda) \frac{(\theta_L R)^2}{2c a^2} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right],
\]

where \( a = 1 + \frac{\eta}{1 - \eta} \frac{H_{HL}}{H_{LL}} (1 - \lambda) \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right] > 1 \). Analogue to the previous section, the first term in both equations is the firm value net of effort cost. The second term, \( \hat{\pi} = \frac{1 - H_{HH}}{h_{HH}} \) is the critical value at which the takeovers occur. The third term in \( p_L^* \) represents the reduction in the takeover price which is proportional to the information rent.

The optimal wages \( \hat{\omega}_i \) are pinned down by the PCL and ICH:

\[
\hat{\omega}_L = \frac{(\theta_L R)^2}{2 c a^2}, \quad (A.2)
\]

\[
\hat{\omega}_H = \frac{(\theta_H R)^2}{2 c} + \frac{H_{HL}}{H_{HH}} \frac{H_{HL}}{h_{HH}} (1 - \lambda) \frac{(\theta_L R)^2}{2 c a^2} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right]. \quad (A.3)
\]

Hence, we end up with a system of equations where \( \hat{e}_L, \hat{p}_L \) and \( \lambda \) are endogenous. However, to prove the proposition we only need to show that \( \lambda \) is a positive quantity. Observing the equation A.1, we know that first two ratios are strictly positive and last term in parenthesis is positive form the definition of \( \theta \) which implies that the monitoring precision is positive.
as long as the effort in the required effort level is positive, which is the case in equilibrium.

Q.E.D.

**Proof of Proposition 7**

Substituting equation A.2 into the equation A.1, yields the following implicit function

$$G(\lambda, \frac{H_{HL}}{H_{LL}}) = \lambda \left[ 1 + A(1 - \lambda) \frac{H_{HL}}{H_{LL}} \right]^2 - A \frac{(\theta_L R)^2 H_{HL}}{2KC H_{LL}}$$

where

$$A = \frac{\eta}{1-\eta} \left[ 1 - \left( \frac{\theta_L}{\theta_H} \right)^2 \right].$$

Assume that an interior solutions exists in both arguments. Note that the function $G(\cdot)$ is cubic in $\lambda$, with positive leading coefficient which implies that it is increasing from bellow. If the value of the function is negative at $\lambda = 0$, then it must cross the $\lambda$-axis from bellow. Since this satisfies the optimality condition $G = 0$, it must be that then the $\frac{\partial G}{\partial \lambda} |_{\lambda^*} > 0$. To verify,

$$G(0, \frac{H_{HL}}{H_{LL}}) = -A \frac{(\theta_L R)^2 H_{HL}}{2KC H_{LL}} < 0,$$

and

$$\frac{\partial G}{\partial \lambda} \bigg|_{\lambda=0} = \left[ 1 + A \frac{H_{HL}}{H_{LL}} \right]^2 > 0.$$ 

Hence, we have shown that the function is bellow the horizontal axis and it is increasing, which proves the conjecture.

Next, note that $G(\cdot)$ is quadratic in the ratio $\frac{H_{HL}}{H_{LL}}$ with positive leading coefficient. By analogy, $G(\cdot)$ must be convex in the ratio, and if the function value is positive at $\frac{H_{HL}}{H_{LL}} = 0$, then the function must cross the horizontal axis from above. Evaluating

$$G(\lambda, 0) = \lambda > 0,$$
by Proposition 6, and re-arranging the equation A.4 yields

\[
G(\lambda, \frac{H_{HL}}{H_{LL}}) = A^2 \lambda(1 - \lambda)^2 \left[ \frac{H_{LL}}{H_{HL}} \right]^2 + \left[ 2\lambda(1 - \lambda) - \frac{(\theta_L R)^2}{2\kappa c} \right] A \frac{H_{HL}}{H_{LL}} + \lambda \frac{H_{LL}}{H_{HL}}, \tag{A.4}
\]

since the coefficient of the linear term in the equation A.4 must be negative, otherwise \( G \) will not have any positive roots. Hence,

\[
\left. \frac{\partial G}{\partial \frac{H_{HL}}{H_{LL}}} \right|_{\frac{H_{HL}}{H_{LL}} = 0} = 2A \lambda(1 - \lambda) - A \frac{(\theta_L R)^2}{2\kappa c} < 0.
\]

Note that the sign of the quantity in the derivative is also implied by the optimality condition which is obvious from inspecting the equation A.4, i.e., by setting \( G = 0 \) and rearranging the terms. To determine the sign, by the Implicit Function Theorem we have

\[
\frac{d\lambda}{d\frac{H_{HL}}{H_{LL}}} = -\frac{\frac{\partial G}{\partial \frac{H_{HL}}{H_{LL}}}}{\frac{\partial G}{\partial \lambda}} > 0.
\]

When \( H \) is log-convex, \( \frac{\partial \frac{H_{HL}}{H_{LL}}}{\partial p_L} < 0 \), however, \( H_{HH} \) remains unaffected since it is not a function of \( p_L \). Specifically, since \( \frac{\partial H_{HL}}{\partial p_L} > 0 \) and \( H_{HH} \) remains fixed, with log-convexity \( \frac{\partial \frac{H_{HL}}{H_{HH}}}{\partial p_L} > 0 \) which is the disciplinary effect on the manager. Thus, an a decrease in \( p_L \) implies higher likelihood of takeovers, lower \( \frac{H_{HL}}{H_{HH}} \), higher \( \frac{H_{HL}}{H_{LL}} \) and, therefore, a higher \( \lambda \) which implies complementarity. Exact opposite holds with log-concavity. Q.E.D.
References


Burkart, Mike, and Fausto Panunzi, 2006, Takeovers, *ECGI-Finance working paper*.


